OVERMIER

A Study of the Building up of a Direct Current Generator

Electrical Engineering

M. S.

1912

A STUDY OF THE BUILDING UP OF A DIRECT CURRENT GENERATOR

BY

MELVERN D. OVERMIER

B. S., University of Illinois, 1911

THESIS

Submitted in Partial Fulfillment of the Requirements for the

Degree of

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IN ELECTRICAL ENGINEERING

IN

THE GRADUATE SCHOOL

OF THE

UNIVERSITY OF ILLINOIS

1 9 1 2

UNIVERSITY OF ILLINOIS

THE GRADUATE SCHOOL

May 31, 1912

I HEREBY RECOMMEND THAT THE THESIS PREPARED UNDER MY SUPERVISION BY

MELVERN D. OVERMIER

ENTITLED A STUDY OF THE BUILDING UP OF A DIRECT CURRENT GENERATOR

BE ACCEPTED AS FULFILLING THIS PART OF THE REQUIREMENTS FOR THE DEGREE OF MASTER OF SCIENCE IN ELECTRICAL ENGINEERING

Ernst Berg
In Charge of Major Work

Ernst Berg
Head of Department

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Ellery B. Paine
J. M. Bryant

Committee on Final Examination

1912
OV2

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A STUDY OF THE BUILDING UP OF A DIRECT CURRENT GENERATOR

--- I INTRODUCTION ---

It is the purpose of this thesis to study the transient effects that follow when direct current voltages are impressed upon inductive circuits containing iron. This is to be occomplished both from a theoretical and experimental point of view. As a special application of the problem, the building up of the field of a direct current generator will be studied. The constants of the machine will be taken and the saturation curve determined. These values will then be substituted into the "step by step" equation and the points actually calculated. These calculated values will then be checked experimentally by means of the oscillograph.

In the following "General Theory", a brief discussion will be given as to the causes of the continuous current transient. From this, a comparison will be made of air and iron cored circuits and their resultant effects upon the building up of the current when the switch is closed on the circuit. Lastly mathematical laws will be deduced showing the relation between time and current at any instant.

--- II GENERAL THEORY ---

During the normal or steady condition in any continuous current circuit, the voltage current and resistance vary according to Ohm's law. That is; the current equals the voltage divided by the resistance. If, however, a change takes place in the circuit conditions as for instance of the impressed voltage or resistance, the change is followed, not be an instantaneous change of the current, but by a transient condition which may be expressed by some exponential law into which the inductance,as well as the resistance, of the circuit enters.

The fundamental conception of an electric circuit is that current can not flow without setting up an electromagnetic field at right angles to the current carrying conductor. The current, in reality, is a measure of this electromagnetic field. As time is required to carry on any process where work is involved, accordingly, time is required to start or alter the electromagnetic field about an electric circuit. Hence time is required to start the electric current or to alter it when once established. There are three quantities that determine the rate of change of the current and the magnetic field in a continuous current circuit, the inductance, resistance and impressed voltage. The latter, however, does not apply to air cored coils.

Now, in considering problems of this type, there are two distinct circuit conditions to be met, first; circuits

that contain iron and second; circuits that do not. Considering for the time the latter, the inductance of such a circuit is a constant value due to the constant value of permeability or magnetic conductivity of air. Inductance is defined as the number of interlinkages of an electric circuit with the lines of magnetic force of the flux produced by unit current in the circuit, that is;

$$L = \frac{N\,\varphi}{10^8\,i} \qquad (1)$$

where L is the self inductance in henries, N the total number of turns, φ the total number of lines of flux and i the current. Thus, it is seen that the self inductance L is dependent upon the flux φ. Now, in air, the permeability or magnetic conductivity is a constant value, that is; unit increase in the magnetizing force produces unit increase in the flux. Thus it is seen from equation No. (1) that the self inductance remains constant for a coil with an air core. The mathematical relation for the current at any instant after closing the switch, in this case, will be developed and discussed later.

Considering now the case of a circuit with a coil containing iron, the self inductance is not a constant value but variable due to the variable magnetic conductivity of iron. Equal increments in the magnetizing force do not produce equal increments in the flux set up in the iron core. A point is finally reached where a large increase in the current produces very little increase in the flux. That is;

the ratio of the flux set up to the current flowing is a decreasing value as the current increases. Hence, referring to equation No. (1), it is seen that the inductance decreases in the same ratio. Thus, it is seen that the value of the current at any instant after closing the switch will be represented by a different law for air and for iron circuits. At the same time that the magnetic conductivity is a variable value for iron it also is a very much higher value for iron. Hence, the inductance will be very much higher than for air cored coils and it should be expected that the rate of change of the flux for iron cored coils, at least at high densities, to be lower than for air cored.

Considering now the mathematics of the continuous current circuit during the transient condition, at the first instant after closing the switch a small current begins to flow. This current sets up a flux which in turn sets up a counter E. M. F. which may be expressed as follows:

$$e = -N\frac{d\varphi}{dt} = -N\frac{d\varphi}{di}\cdot\frac{di}{dt} = -L\frac{di}{dt}$$

By Lentz's law, the expression is negative, since it is opposed to the impressed E. M. F. Then, if E equals the impressed E. M. F., R the resistance, d i the increase of current during time moment d t and L the inductance, then the following expression may be written:

$$E = i\,R + L\,\frac{d\,i}{d\,t}$$

Transposing, $\frac{d\,i}{d\,t} + \frac{i\,R}{L} = \frac{E}{L}$

This is of the type form;

$$\frac{d\,y}{d\,x} + A\,y = B$$

Whence $y = \varepsilon^{-\int A\,d\,x}\left[\int \varepsilon^{+\int A\,d\,x} . B\,d\,x + C\right]$

where $d\,x = d\,t,\quad y = i,\quad A = \frac{r}{L},\quad B = \frac{E}{L}$

Then $i = \varepsilon^{-\frac{R}{L}t}\left[\int \varepsilon^{+\frac{R}{L}t} . \frac{E}{L}\,d\,t + C\right]$

$$i = \varepsilon^{-\frac{R}{L}t}\left[\frac{E}{L}\int \varepsilon^{\frac{R}{L}t}\,d\,t + C\right]$$

$$= \varepsilon^{-\frac{R}{L}}\left[\frac{E}{\cancel{L}} . \frac{\cancel{L}}{R}\,\varepsilon^{\frac{R}{L}t} + C\right]$$

At the instant of closing the switch when

$$t = 0, \text{ then } i = 0$$

Then, $C = -\frac{E}{R}$

Then, $i = \frac{E}{R} - \frac{E}{R}\cdot\varepsilon^{-\frac{R}{L}t}$

or $i = \frac{E}{R}\left[1 - \varepsilon^{-\frac{R}{L}t}\right]$

This equation which is fairly simple is the value of the current at any instant t after closing the switch. This equation, however, does not take into account the saturation curve. The case here assumed is that of a straight line saturation curve such as we would get from an air cored coil.

Consider now the case of an iron cored coil where

the saturation curve is taken into account. The latter is the case of the direct current field on which the oscillograms and calculations for this thesis were based. In 1882 Frölich developed an empirical formula for the saturation curve which can, approximately, as was first shown, be represented by a parabolic curve.

$$\Phi = \frac{K\ i}{1 + K_1\ i}$$

Where Φ is the magnetic flux, K and K_1 are constants which depend upon the ratio of the flux developed to the current flowing. i is the current flowing in the circuit. If we know two simultaneous values of Φ and the corresponding values of i, then we may solve simultaneously and get the values of K and K_1 for the particular saturation curve.

Substituting this equation in the equation for the self inductance, we get;

$$L = \frac{N\ \Phi}{10^8\ i} = \frac{N\ K}{10\ \ (1 + K_1\ \ i)}$$

Then we have

$$E = i\ r + L\ \frac{d\ i}{d\ t} = i\ r + \frac{N\ K}{10^8 (1 + K_1\ i\ )}\ .\ \frac{d\ i}{d\ t}$$

Separating the variables

$$(E - i\ r)\ (1 + K_1\ i) = \frac{N\ K}{10^8}\ .\ \frac{d\ i}{d\ t}$$

$$\frac{10^8\ d\ t}{N\ K} = \frac{d\ i}{(E - i\ r)\ (1 + K_1\ i)}$$

Then,
$$\int \frac{d\,i}{(E - i\,r)\,(i + K_1\,i)} = \frac{t \times 10^8}{N\,K} + C$$

But we have the following relation;

$$\int \frac{d\,x}{(a + b\,x)\,(a' + b'\,x)} = \frac{1}{a\,b' - a'\,b} \,.\, \log. \frac{a' + b'\,x}{a + b\,x}$$

Then,
$$\int \frac{d\,i}{(E - i\,r)\,(1 + K_1\,i)} = \frac{1}{E\,K_1 + r} \,.\, \log. \frac{1 + K_1\,i}{E - i\,r}$$

$$\frac{1}{E\,K_1 + r} \log. \frac{1 + K_1\,i}{E - i\,r} = \frac{t \times 10^8}{N\,K} + C$$

$$\mathrm{Log.} \frac{1 + K_1\,i}{E - i\,r} = \frac{10^8 t\,(E\,K_1 + r)}{N\,K} + C$$

If we have,

$$\mathrm{Log}_{\varepsilon}\, a = b + C$$

Then
$$a = \varepsilon^{b + C} = \varepsilon^{b} + C_1 = C_2 \cdot \varepsilon^{b}$$

Then
$$\frac{1 + K_1\,i}{E - i\,r} = C_2 \cdot \varepsilon^{-\frac{10^8\,(E\,K_1 + r)\,t}{N\,K}}$$

The starting conditions are,

for $t = 0$, $i = 0$

Then
$$C_2 = \frac{1}{E}$$

and
$$\frac{1 + K_1\,i}{E - i\,r} = \frac{1}{E} \cdot \varepsilon^{\frac{10^8\,(E\,K_1 + r)}{N\,K} t}$$

Let $$\frac{10^8 (E K_1 + r)}{N K} = P$$

Then $$i = -\frac{E (1 - \varepsilon^{P t})}{K_1 E + r \cdot \varepsilon^{P t}}$$

This formula gives a relation between the current and time at any instant after closing the switch for iron cored coils and is correct within the limits of the saturation curve.

While it is a fairly simple matter to develop an equation for the current at any instant in air cored coils and is possible to derive a differential equation that may be integrated for iron circuits, yet it is a much simpler matter, in the latter case, to develop a "step by step" method in which values from the actual saturation curve are used.

We have, $$E = i\ r + \frac{N}{10^8} \cdot \frac{d\ \varphi}{d\ t}$$

Now take increments instead of differentials

Then $$E - i\ r = \frac{N}{10^8} \cdot \frac{\Delta \varphi}{\Delta t}$$

Assume $E = 100$ volts

$r = 100$ ohms

$N = 4000$ turns

and take increments of 0.1 Sec. Then $\Delta t = 0.1$

Then $$E - i\ r = \frac{N}{10^7} \cdot \Delta \varphi = 100 - 100\ i = 100\ (1 - i)$$

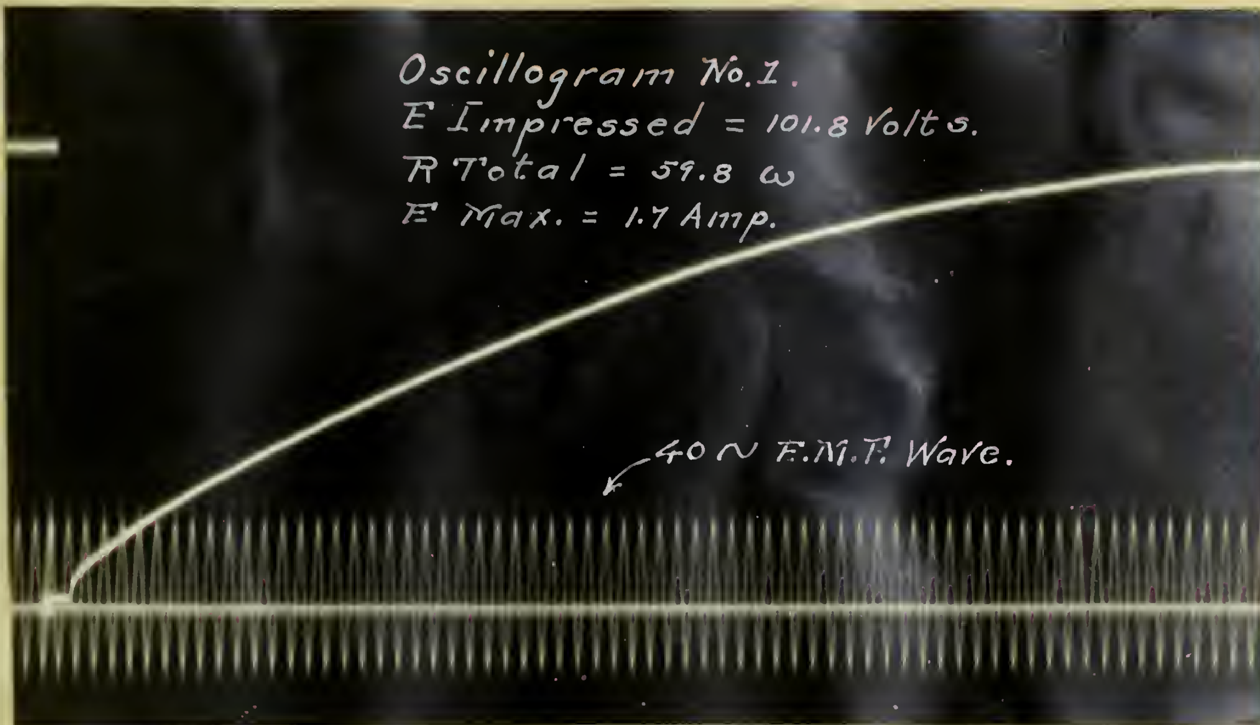
Then $$\Delta \varphi = 0.25 \times 10^6\ (1 - i)$$

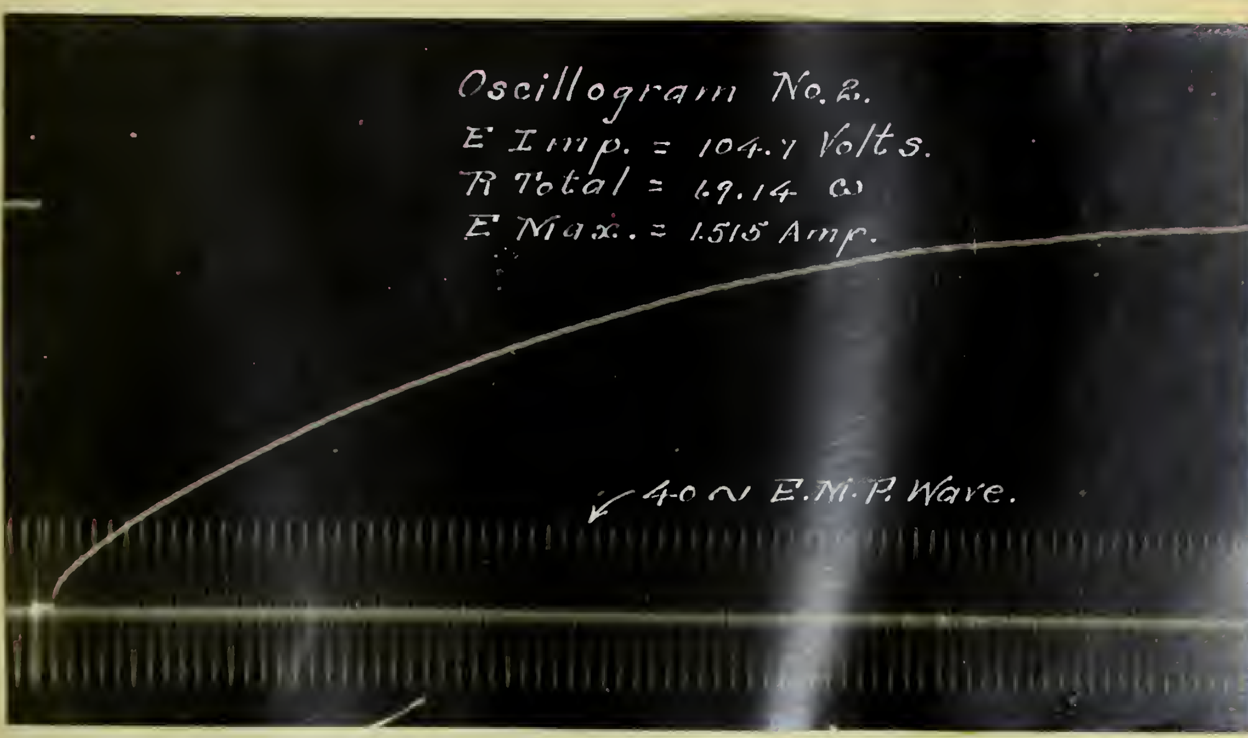
$$\Phi = \Sigma \Delta\ \varphi$$

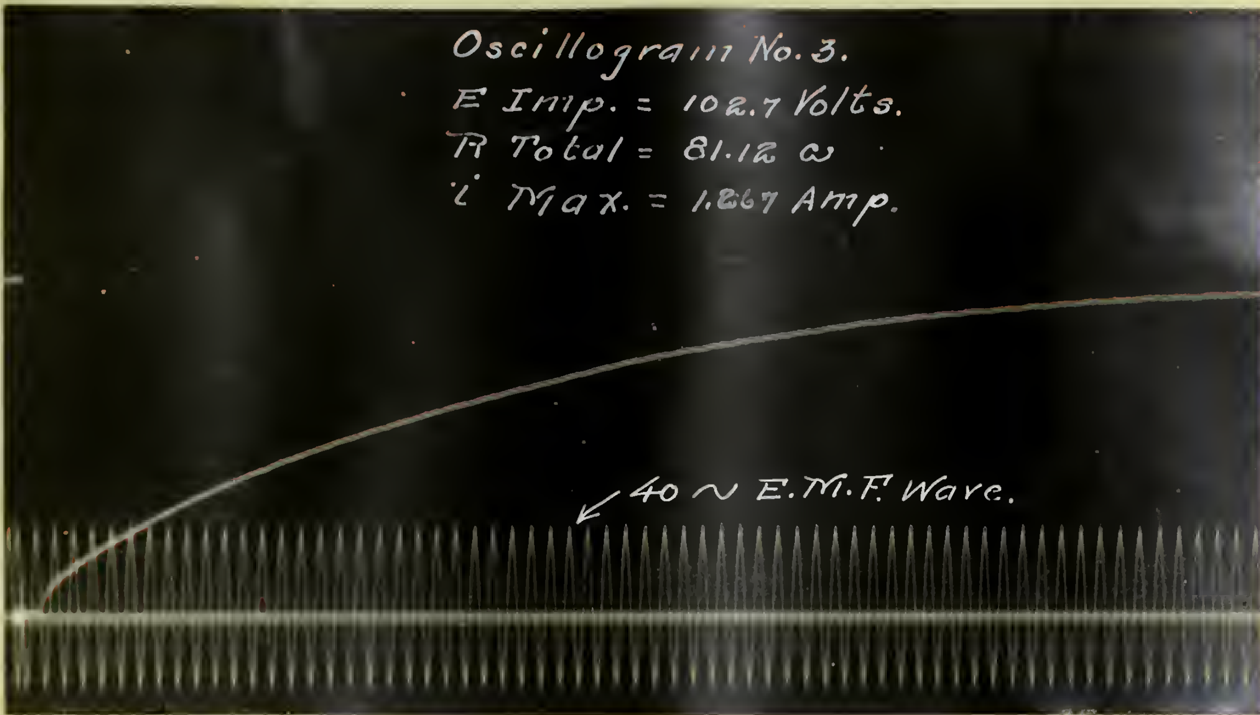
--- III DESCRIPTION OF TESTS ---

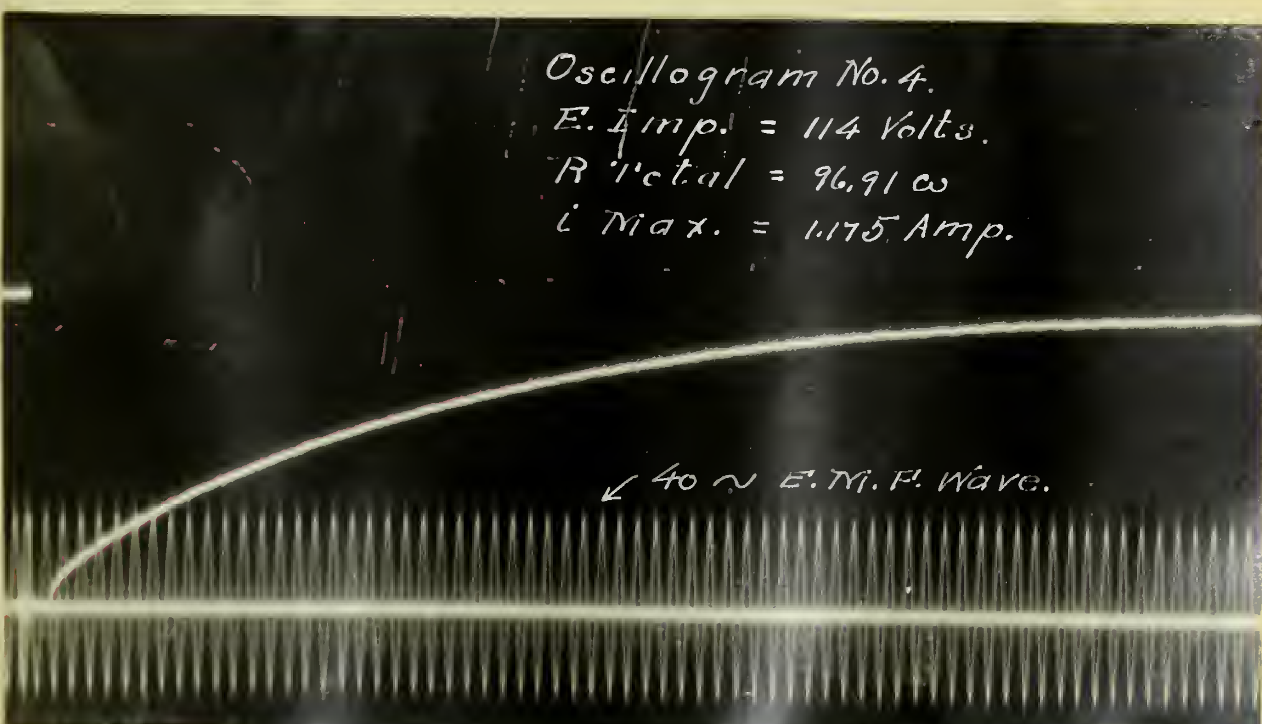
First of all, an attempt was made to check the theoretical values by placing an ammeter directly in series with the field of the generator, and by means of an induction coil and a contact making device, causing a known number of sparks per second to pierce a thin piece of paper between the needle point and the metal plate below. Then, from this series of perforations, made as the needle swung to its final value, the current at definite intervals could be read. This device, however, proved a failure due to the inertia of the moving element of the ammeter index. It then became necessary to check the theoretical values entirely by the oscillograph. The generator and oscillograph were placed close together and the power drawn from the substation close at hand in order to make the line resistance inappreciable. The voltage impressed on the field of the machine was read just at the instant before closing the switch on the oscillograph. The resistance of the field and rheostat was measured. Then, from the voltage and resistance readings the current was calculated. Eight different oscillograms were taken; four with 110 volts and four with 220 volts impressed. The first oscillogram for each voltage was taken with the field resistance alone. Then, successively, for the 110 volt readings approximately 10, 20 and 30 ohms were included in the circuit; for the 220 volt readings, approximately 15, 30 and 60 ohms were included.

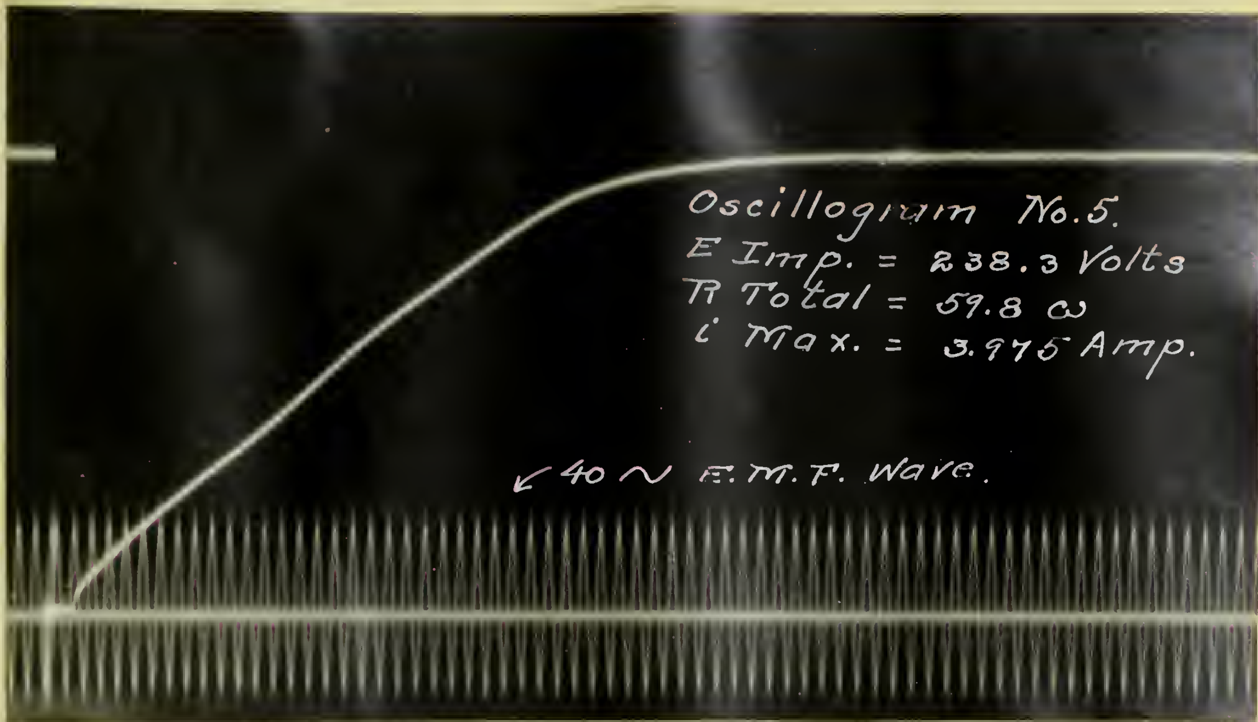
Before taking the data for the saturation curve, an attempt was made to discharge the field of its residual magnetism by reversing a current back and forth through the windings and gradually cutting in resistance until the current was brought down practically to zero. The data for the curve was then very carefully taken. A separate motor-generator set was run to supply the voltage to the motor which drove the generator under test. Another set was run to supply the voltage to the field of the generator. Both these voltages were kept constant. Speed tests also were constantly run on the generator whose saturation curve was being taken. From the voltage generated, and the constants of the machine, the flux was figured corresponding to any particular value of the field current.

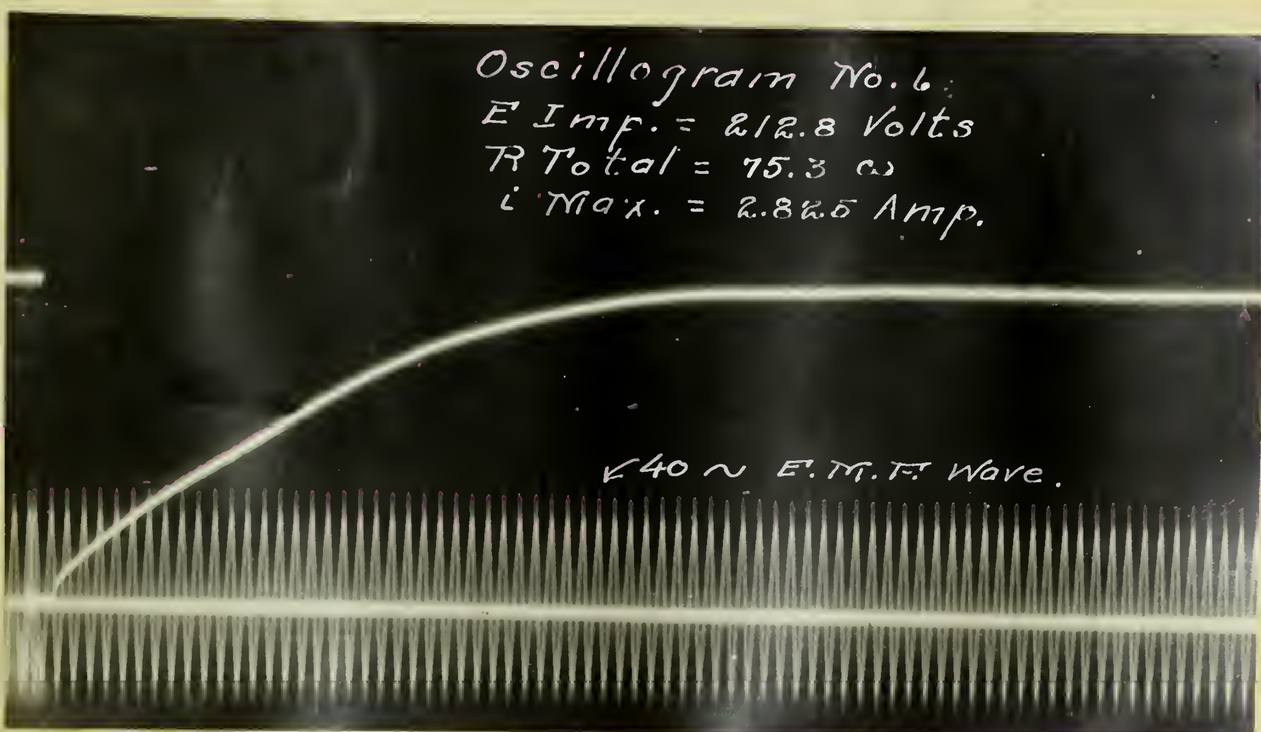

Oscillogram No. 1.
E Impressed = 101.8 Volts.
R Total = 59.8 ω
E Max. = 1.7 Amp.
40 ~ E.M.F. Wave.

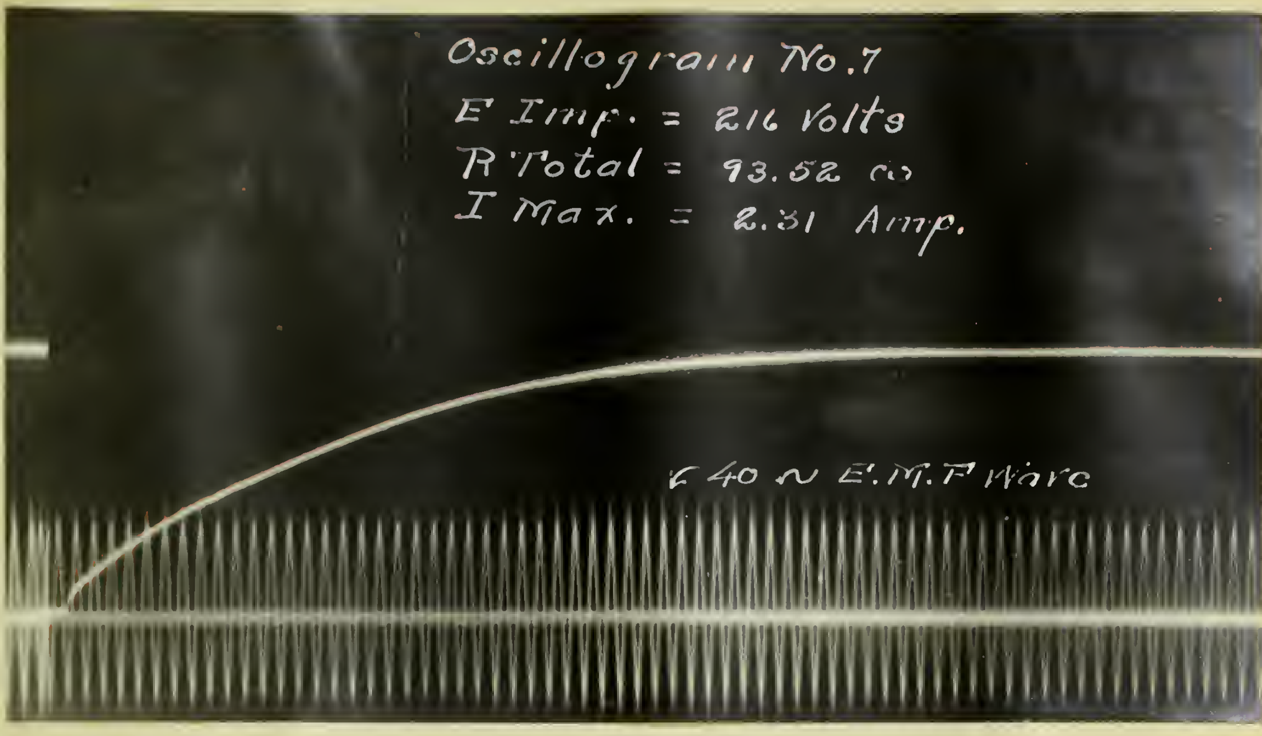

Oscillogram No. 2.
E Imp. = 104.7 Volts.
R Total = 69.14 ω
E Max. = 1.515 Amp.
40 ~ E.M.F. Wave.

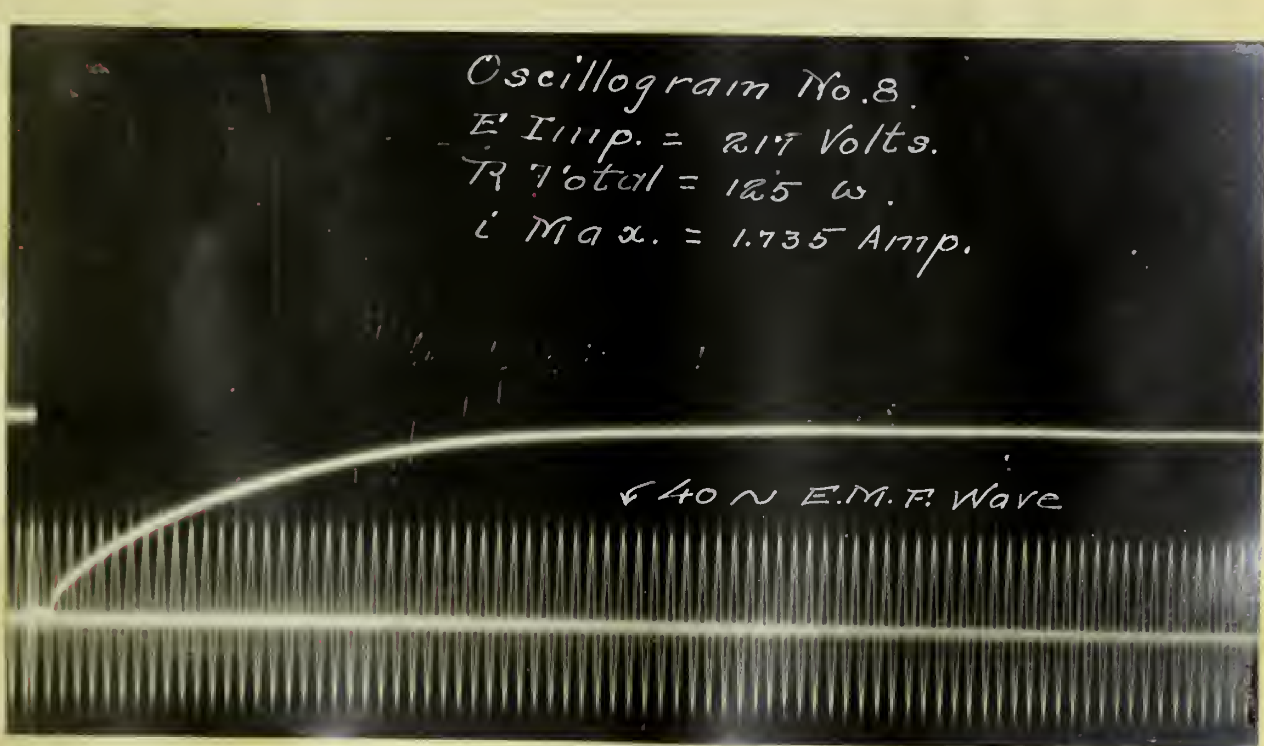

Oscillogram No. 3.
E Imp. = 102.7 Volts.
R Total = 81.12 ω
i Max. = 1.267 Amp.
40 ∼ E.M.F. Wave.

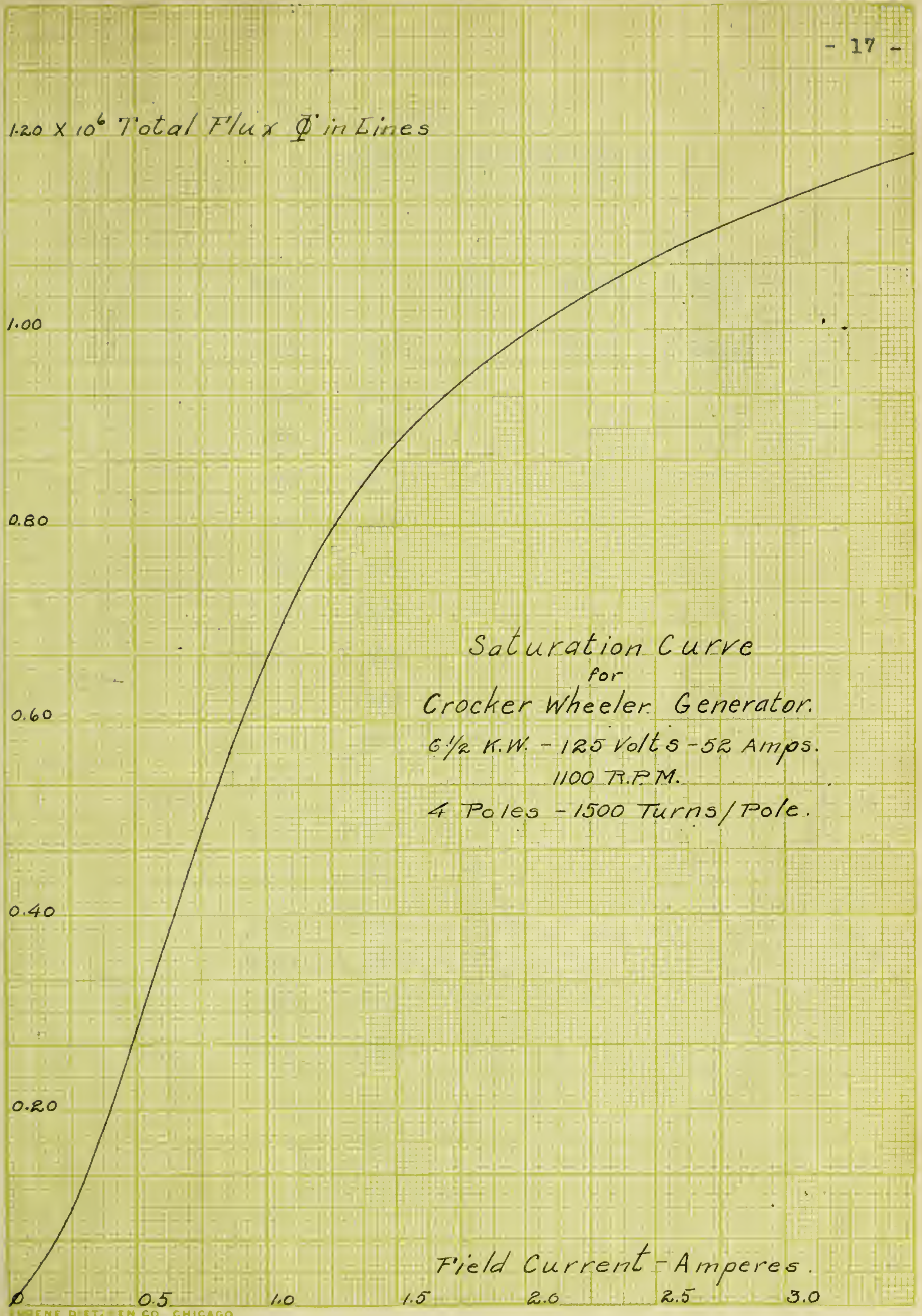

Oscillogram No. 4.
E. Imp. = 114 Volts.
R Total = 96.91 ω
i Max. = 1.175 Amp.
40 ∼ E.M.F. Wave.


Oscillogram No. 5.
E Imp. = 238.3 Volts
R Total = 59.8 ω
i Max. = 3.975 Amp.
40 ~ E.M.F. Wave.


Oscillogram No. 6.
E Imp. = 212.8 Volts
R Total = 75.3 ω
i Max. = 2.825 Amp.
40 ~ E.M.F. Wave.


Oscillogram No.7
E Imp. = 216 Volts
R Total = 93.52 ω
I Max. = 2.31 Amp.
40 ~ E.M.F Wave


Oscillogram No.8.
E Imp. = 217 Volts.
R Total = 125 ω.
i Max. = 1.735 Amp.
40 ~ E.M.F. Wave


1.20 X 10^6 Total Flux Φ in Lines
1.00
0.80
0.60
0.40
0.20
0
0.5
1.0
1.5
2.0
2.5
3.0
Field Current - Amperes.
Saturation Curve
for
Crocker Wheeler Generator.
6½ K.W. - 125 Volts - 52 Amps.
1100 R.P.M.
4 Poles - 1500 Turns/Pole.
EUGENE DIETZGEN CO., CHICAGO.

Data From Oscillogram No. 1.

t sec.	i	R ex.	R total	E imp.
.05	.190	0	59.8	101.8
.10	.280	"	"	"
.20	.440	"	"	"
.40	.715	"	"	"
.60	.950	"	"	"
.80	1.167	"	"	"
1.00	1.350	"	"	"
1.20	1.485	"	"	"
1.40	1.565	"	"	"
1.60	1.630	"	"	"
1.80	1.665	"	"	"
2.00	1.682	"	"	"
2.20	1.690	"	"	"
2.40	1.695	"	"	"
2.60	1.697	"	"	"
2.80	1.700	"	"	"
3.00	1.700	"	"	"

Calculated Data, Oscillogram No. 1.

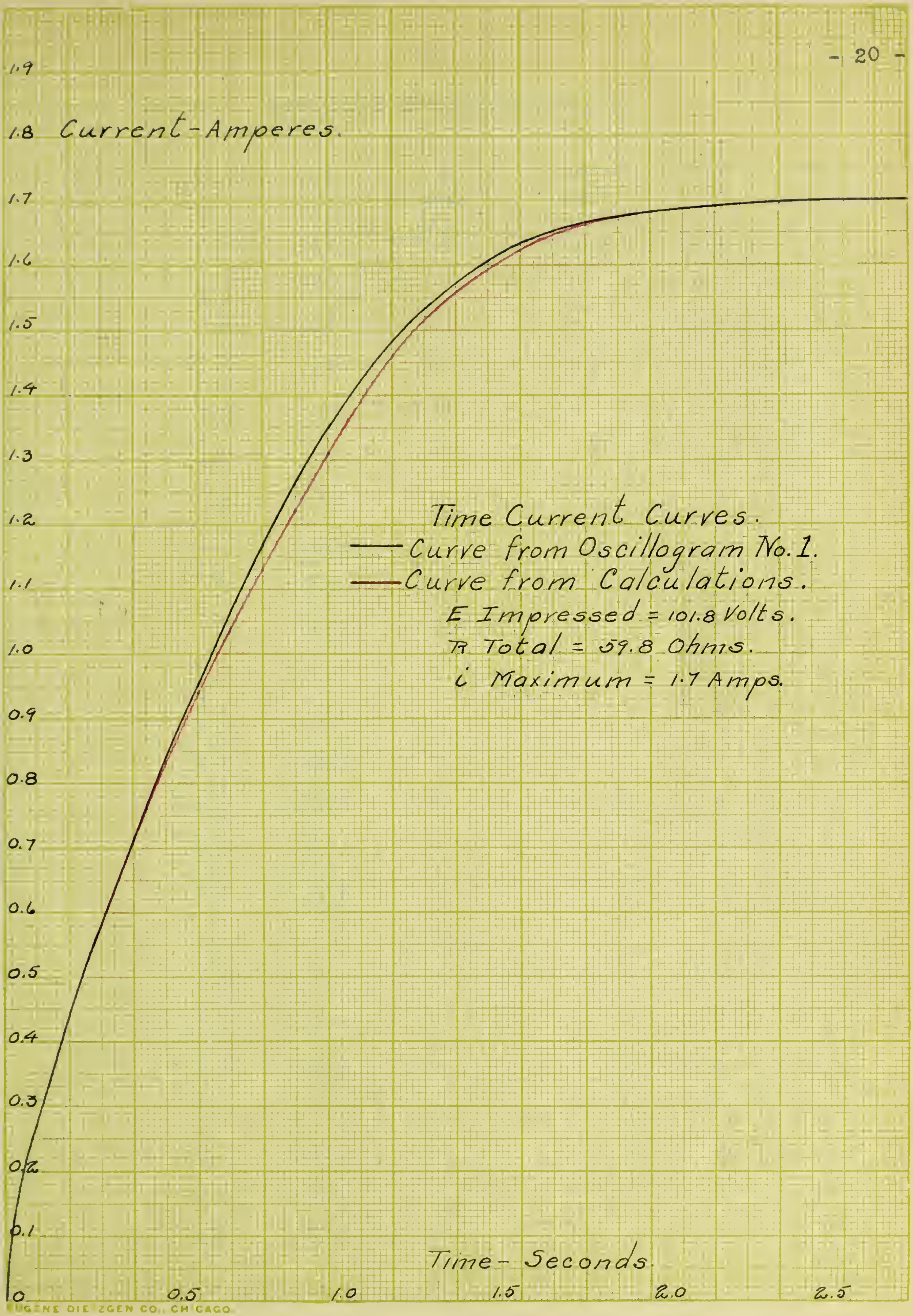
E = 101.8 Volts R = 59.8 ohms N = 6000 turns

Assume $\Delta t = 0.1$ sec.

$$\text{Then } E - iR = \frac{N}{10^7}\cdot\Delta\varphi = 101.8 - 59.8\, i = 59.8\,(1.702 - i)$$

$$\Delta\varphi = \frac{59.8 \times 10^7}{6000}(1.702 - i) = .09975 \times 10^6\,(1.702 - i)$$

t	$\Delta\varphi$	$\Sigma\Delta\varphi$	i	(1.702 - i)	$\Delta\varphi$	$\Sigma\Delta\varphi$	i
.1	.170	.170	.36	1.342	.1339	.1339	.31
.2	.1339	.268	.48	1.222	.1220	.2559	.47
.3	.1220	.378	.62	1.082	.1079	.3638	.60
.4	.1079	.472	.73	.972	.0969	.4607	.72
.5	.0969	.558	.85	.852	.0849	.5456	.83
.6	.0849	.630	.95	.752	.0750	.6206	.94
.7	.0750	.696	1.07	.632	.0630	.6836	1.04
.8	.0630	.747	1.16	.542	.0540	.7376	1.13
.9	.0540	.792	1.25	.452	.0451	.7827	1.22
1.0	.0451	.828	1.33	.372	.0371	.8198	1.31
1.1	.0371	.857	1.41	.292	.0291	.8489	1.39
1.2	.0291	.878	1.48	.222	.0221	.8710	1.46
1.3	.0221	.893	1.53	.172	.0171	.8881	1.52
1.4	.0171	.905	1.58	.122	.0122	.9003	1.56
1.5	.0122	.913	1.61	.092	.0092	.9095	1.60
1.6	.0092	.919	1.63	.072	.0072	.9167	1.62
1.7	.0072	.924	1.66	.042	.0042	.9209	1.63
1.8	.0042	.925	1.67	.032	.0032	.9241	1.66
1.9	.0032	.927	1.68	.022	.0022	.9263	1.67



Current-Amperes.
1.9
1.8
1.7
1.6
1.5
1.4
1.3
1.2
1.1
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0.5
1.0
1.5
2.0
2.5
Time Current Curves.
Curve from Oscillogram No. 1.
Curve from Calculations.
E Impressed = 101.8 Volts.
R Total = 59.8 Ohms.
i Maximum = 1.7 Amps.
Time- Seconds
EUGENE DIETZGEN CO., CHICAGO

Data from Oscillogram No. 2.

t	i	R ex.	R total	E
.05	.199	9.34	69.14	104.7
.1	.290	"	"	"
.2	.447	"	"	"
.3	.579	"	"	"
.4	.706	"	"	"
.5	.815	"	"	"
.6	.912	"	"	"
.7	1.005	"	"	"
.8	1.080	"	"	"
.9	1.159	"	"	"
1.0	1.231	"	"	"
1.1	1.286	"	"	"
1.2	1.327	"	"	"
1.3	1.373	"	"	"
1.4	1.404	"	"	"
1.5	1.421	"	"	"
1.6	1.445	"	"	"
1.7	1.460	"	"	"
1.8	1.475	"	"	"
1.9	1.485	"	"	"
2.0	1.497	"	"	"
2.1	1.520	"	"	"
2.2	1.511	"	"	"
2.3	1.515	"	"	"

Calculated Data, Oscillogram No. 2.

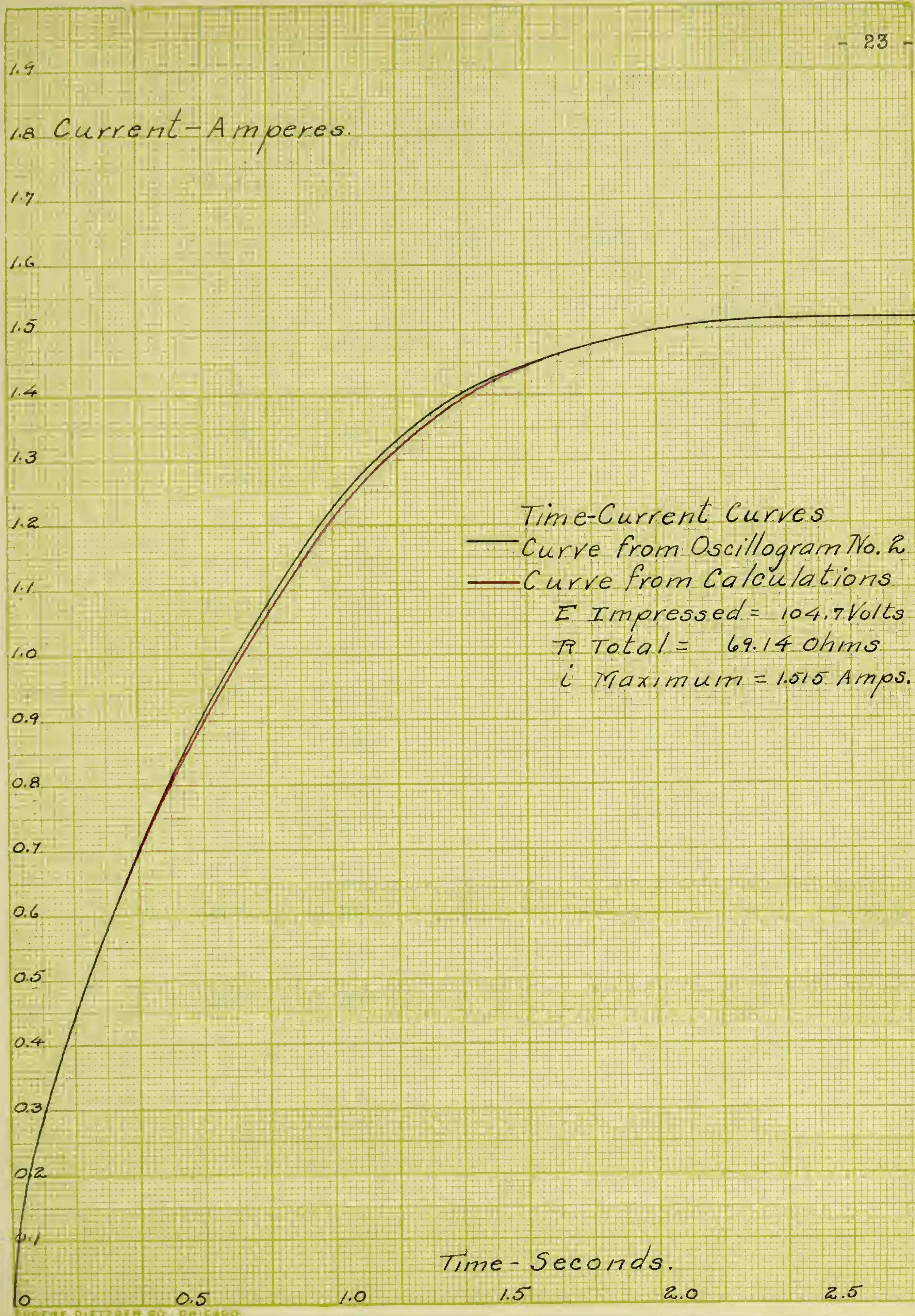
E = 104.7 Volts R = 69.14 ohms N. = 6000 turns

Assume Δt = 0.10 seconds.

Then, $E - iR = \frac{N}{10^7} \cdot \Delta\varphi = 104.7 - 69.14\, i = 69.14\,(1.514 - i)$

$$\Delta\varphi = \frac{69.14 \times 10^7}{6000}(1.514 - i) = .1152 \times 10^6\,(1.514 - i)$$

t	$\Delta\varphi$	$\Sigma\Delta\varphi$	i	(1.514 - i)	$\Delta\varphi$	$\Sigma\Delta\varphi$	i
.1	.175	.175	.37	1.144	.1319	.1319	.30
.2	.1319	.264	.48	1.037	.1194	.2513	.46
.3	.1194	.371	.61	.904	.1043	.3556	.59
.4	.1043	.460	.72	.794	.0915	.4471	.70
.5	.0915	.539	.83	.684	.0788	.5259	.81
.6	.0788	.605	.92	.599	.0690	.5949	.90
.7	.0690	.664	1.01	.504	.0582	.6531	.99
.8	.0582	.711	1.08	.434	.0500	.7031	1.07
.9	.0500	.753	1.17	.344	.0396	.7427	1.14
1.0	.0396	.782	1.22	.294	.0339	.7766	1.21
1.1	.0339	.811	1.29	.224	.0258	.8024	1.27
1.2	.0258	.828	1.33	.184	.0212	.8236	1.32
1.3	.0212	.845	1.38	.134	.0154	.8390	1.37
1.4	.0154	.854	1.41	.104	.0120	.8510	1.39
1.5	.0120	.863	1.43	.084	.0097	.8607	1.42
1.6	.0097	.870	1.46	.054	.0062	.8669	1.45
1.7	.0062	.873	1.47	.044	.0051	.8720	1.46
1.8	.0051	.877	1.48	.034	.0039	.8759	1.48
1.9	.0039	.880	1.49	.024	.0028	.8787	1.49


Current-Amperes.
1.9
1.8
1.7
1.6
1.5
1.4
1.3
1.2
1.1
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0.5
1.0
1.5
2.0
2.5
Time-Seconds.
Time-Current Curves
Curve from Oscillogram No. 2
Curve from Calculations
E Impressed = 104.7 Volts
R Total = 69.14 Ohms
i Maximum = 1.515 Amps.
EUGENE DIETZGEN CO., CHICAGO.

Data From Oscillogram No. 3.

t	i	R ex.	R total	E imp.
.05	.19	21.32	81.12	102.7
.10	.281	"	"	"
.2	.434	"	"	"
.4	.660	"	"	"
.6	.805	"	"	"
.8	.968	"	"	"
1.0	1.070	"	"	"
1.2	1.132	"	"	"
1.4	1.177	"	"	"
1.6	1.205	"	"	"
1.8	1.222	"	"	"
2.0	1.240	"	"	"
2.2	1.250	"	"	"
2.4	1.260	"	"	"
2.6	1.260	"	"	"
2.8	1.267	"	"	"
3.0	1.267	"	"	"

Calculated Data, Oscillogram No. 3.

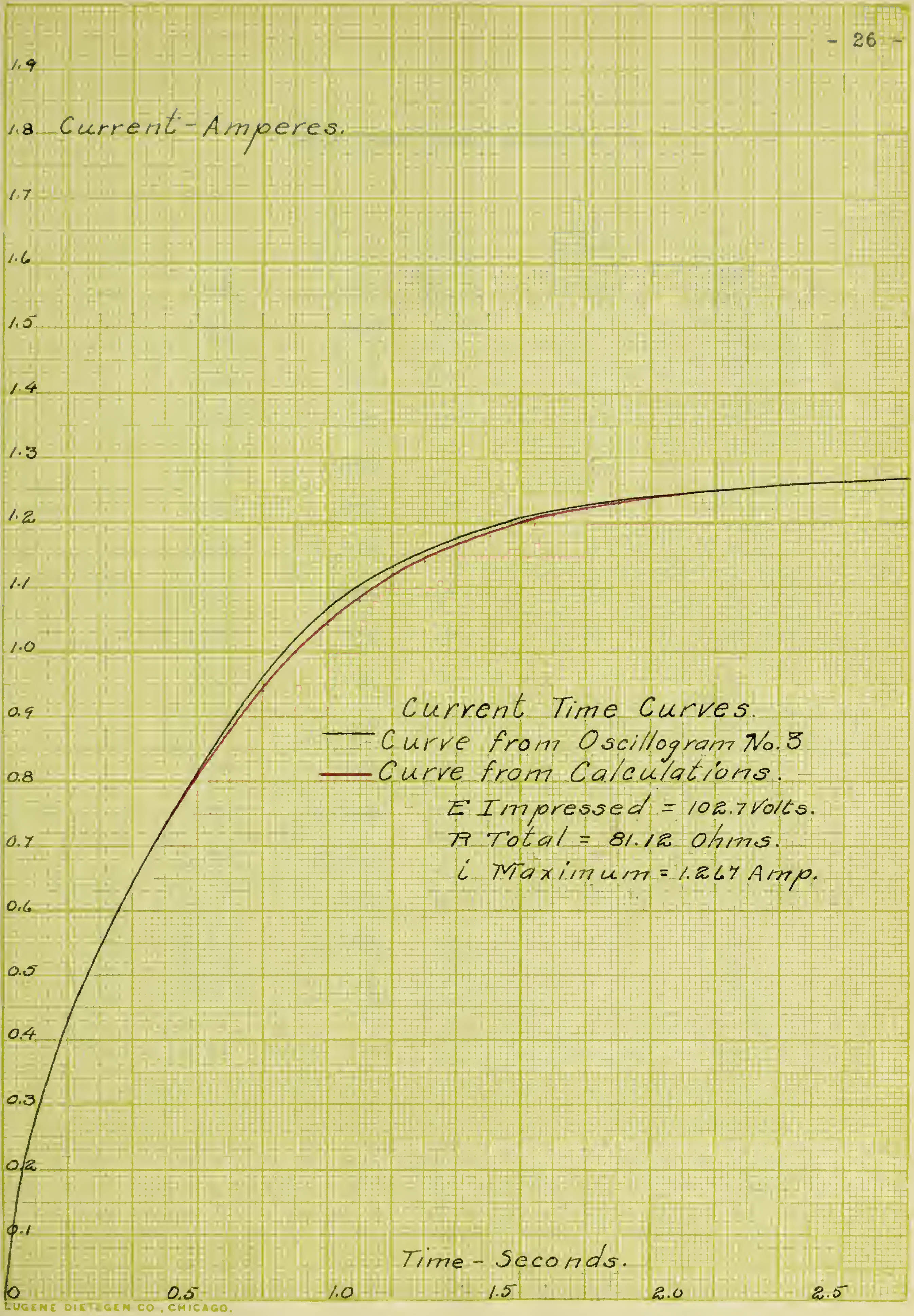
E = 102.7 Volts   R = 81.12 ohms   N = 6000 turns

Assume Δt = 0.1 seconds

Then, $E - iR = \frac{N}{10^7} \cdot \Delta\varphi = 102.7 - 81.12\, i = 81.12\,(1.265 - i)$

$$\Delta\varphi = \frac{81.12 \times 10^7}{6000}(1.265 - i) = .1352 \times 10^6\,(1.265 - i)$$

t	$\Delta\varphi$	$\Sigma\Delta\varphi$	i	(1.265 - i)	$\Delta\varphi$	$\Sigma\Delta\varphi$	i
.1	.171	.171	.36	.905	.1223	.1223	.28
.2	.1223	.245	.46	.805	.1087	.2310	.43
.3	.1087	.340	.57	.695	.0939	.3249	.56
.4	.0939	.419	.67	.595	.0804	.4053	.65
.5	.0804	.486	.76	.505	.0682	.4735	.74
.6	.0682	.542	.83	.435	.0587	.5322	.82
.7	.0587	.591	.90	.365	.0493	.5815	.88
.8	.0493	.631	.95	.315	.0426	.6241	.94
.9	.0426	.667	1.02	.245	.0331	.6572	1.00
1.0	.0331	.690	1.05	.215	.0291	.6863	1.04
1.1	.0291	.715	1.10	.165	.0223	.7086	1.08
1.2	.0223	.731	1.12	.145	.0196	.7282	1.12
1.3	.0196	.748	1.16	.105	.0142	.7424	1.14
1.4	.0142	.757	1.18	.085	.0115	.7539	1.17
1.5	.0115	.765	1.19	.075	.0101	.7640	1.18
1.6	.0101	.774	1.21	.055	.0074	.7714	1.20
1.7	.0074	.779	1.22	.045	.0061	.7775	1.21
1.8	.0061	.784	1.23	.035	.0047	.7822	1.22
1.9	.0047	.787	1.24	.025	.0034	.7856	1.23


Current - Amperes.
1.9
1.8
1.7
1.6
1.5
1.4
1.3
1.2
1.1
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
Current Time Curves.
Curve from Oscillogram No. 3
Curve from Calculations.
E Impressed = 102.7 Volts.
R Total = 81.12 Ohms.
i Maximum = 1.267 Amp.
Time - Seconds.
0.5
1.0
1.5
2.0
2.5
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Data From Oscillogram No. 4.

t	i	R ex.	R total	E
.05	.199	37.11	96.91	114
.10	.281	"	"	"
.20	.443	"	"	"
.40	.652	"	"	"
.60	.805	"	"	"
.80	.923	"	"	"
1.00	1.013	"	"	"
1.20	1.068	"	"	"
1.40	1.104	"	"	"
1.60	1.131	"	"	"
1.80	1.149	"	"	"
2.00	1.154	"	"	"
2.20	1.165	"	"	"
2.40	1.168	"	"	"
2.60	1.172	"	"	"
2.80	1.175	"	"	"
3.00	1.175	"	"	"

Calculated Data, Oscillogram No. 4.

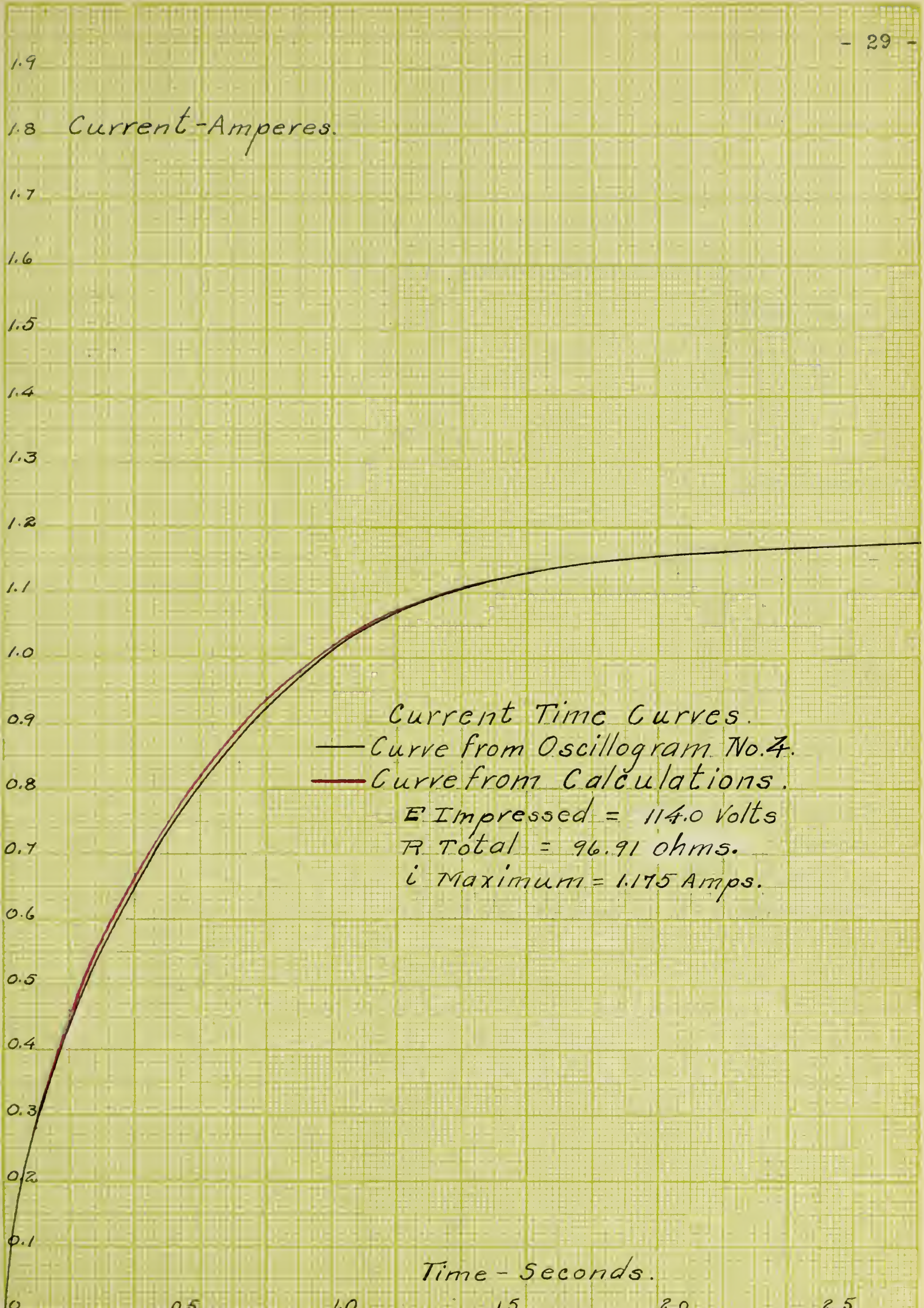
E = 114 Volts R = 96.91 ohms N = 6000 turns

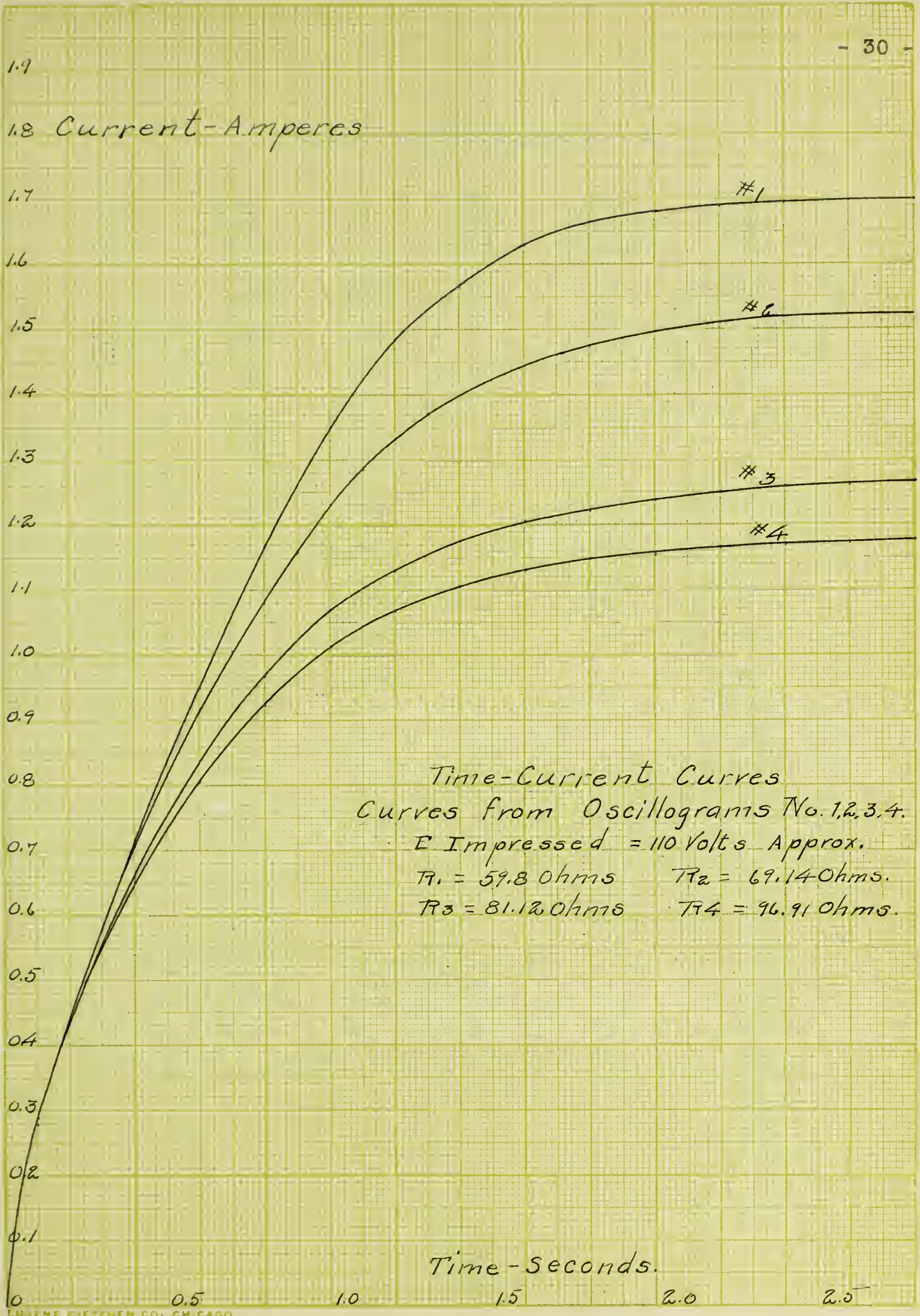
Assume $\Delta t = 0.1$ seconds

Then, $$E - iR = \frac{N}{10^7} \cdot \Delta\varphi = 114 - 96.91\, i = 96.91\,(1.176 - i)$$

$$\Delta\varphi = \frac{96.91 \times 10^7}{6000}\,(1.176 - i) = .1615 \times 10^6\,(1.176 - i)$$

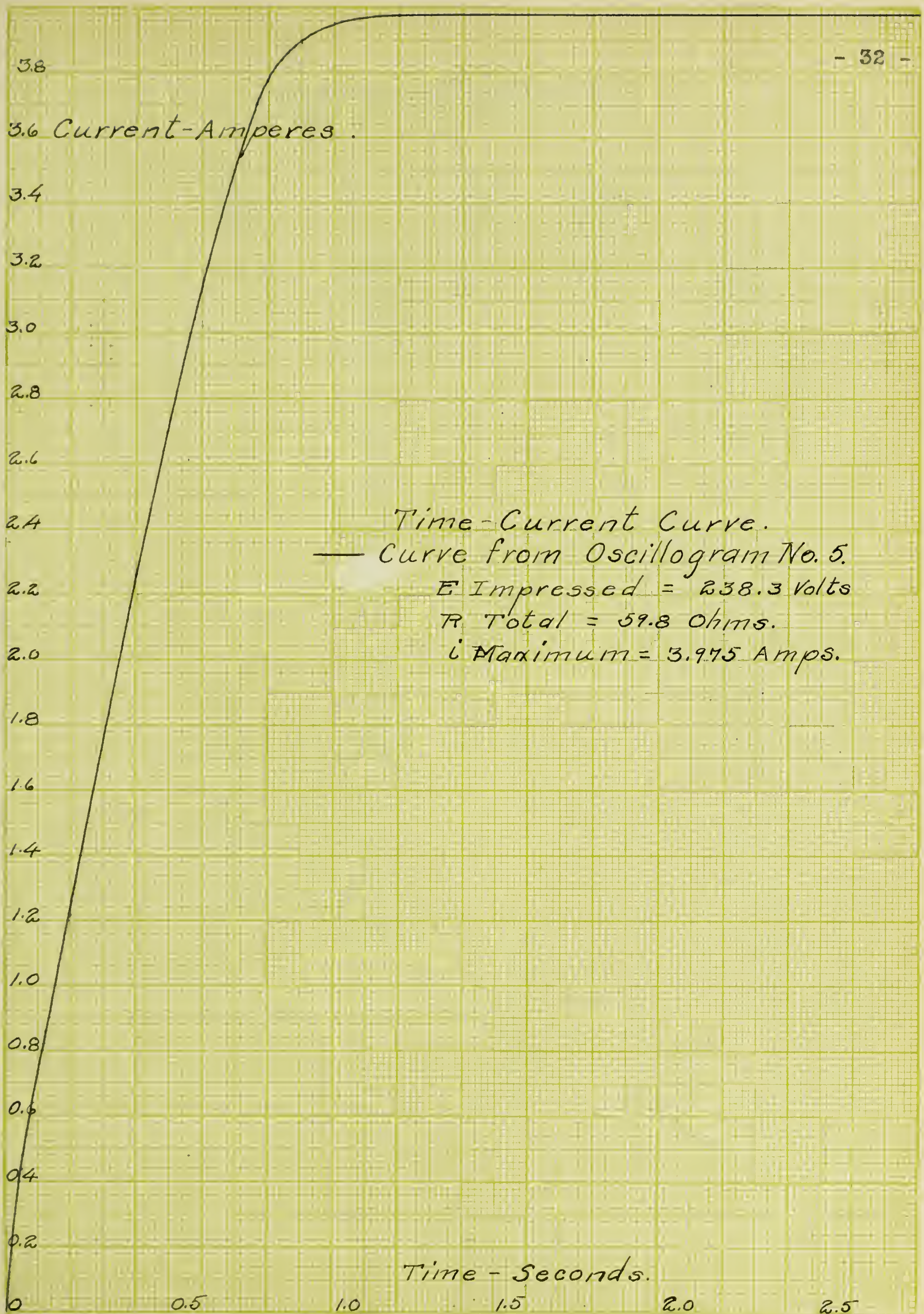
t	$\Delta\varphi$	$\Sigma\Delta\varphi$	i	(1.176 - i)	$\Delta\varphi$	$\Sigma\Delta\varphi$	i
.1	.190	.190	.38	.796	.1285	.1285	.29
.2	.1285	.257	.47	.706	.1140	.2425	.46
.3	.1140	.357	.59	.586	.0946	.3371	.57
.4	.0946	.432	.68	.496	.0800	.4171	.67
.5	.0800	.497	.77	.406	.0655	.4826	.75
.6	.0655	.548	.83	.346	.0559	.5385	.82
.7	.0559	.594	.90	.276	.0446	.5831	.89
.8	.0446	.628	.95	.226	.0365	.6196	.94
.9	.0365	.656	1.00	.176	.0284	.6480	.98
1.0	.0284	.676	1.03	.146	.0236	.6716	1.02
1.1	.0236	.695	1.06	.116	.0187	.6903	1.05
1.2	.0187	.709	1.09	.086	.0139	.7042	1.08
1.3	.0139	.718	1.11	.066	.0107	.7149	1.09
1.4	.0107	.726	1.12	.056	.0090	.7239	1.11
1.5	.0090	.733	1.13	.046	.0074	.7313	1.12
1.6	.0074	.739	1.14	.036	.0058	.7371	1.13
1.7	.0058	.743	1.15	.026	.0042	.7413	1.14
1.8	.0042	.745	1.155	.021	.0034	.7447	1.15
1.9	.0034	.748	1.160	.016	.0026	.7473	1.16


Current-Amperes.
1.9
1.8
1.7
1.6
1.5
1.4
1.3
1.2
1.1
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
Current Time Curves.
Curve from Oscillogram No.4.
Curve from Calculations.
E Impressed = 114.0 Volts
R Total = 96.91 ohms.
i Maximum = 1.175 Amps.
Time - Seconds.
0.5
1.0
1.5
2.0
2.5


Current-Amperes
1.9
1.8
1.7
1.6
1.5
1.4
1.3
1.2
1.1
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
#1
#2
#3
#4
Time-Current Curves
Curves from Oscillograms No. 1,2,3,4.
E Impressed = 110 Volts Approx.
R1 = 59.8 Ohms R2 = 69.14 Ohms.
R3 = 81.12 Ohms R4 = 96.91 Ohms.
Time-Seconds.
0.5
1.0
1.5
2.0
2.5

Data From Oscillogram No. 5.

t sec.	i	R ex.	R total	E imp.
.05	.439	0	59.8	238.3
.10	.732	"	"	"
.20	1.215	"	"	"
.40	2.260	"	"	"
.60	3.200	"	"	"
.80	3.788	"	"	"
.90	3.892	"	"	"
1.00	3.953	"	"	"
1.2	3.975	"	"	"
1.4	3.975	"	"	"
1.6	"	"	"	"
1.8	"	"	"	"
2.0	"	"	"	"
2.2	"	"	"	"
2.4	"	"	"	"
2.6	"	"	"	"
2.8	"	"	"	"
3.0	"	"	"	"


3.8
3.6 Current-Amperes.
3.4
3.2
3.0
2.8
2.6
2.4
2.2
2.0
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0
Time-Current Curve.
Curve from Oscillogram No. 5.
E Impressed = 238.3 Volts
R Total = 59.8 Ohms.
i Maximum = 3.975 Amps.
Time - Seconds.
0.5
1.0
1.5
2.0
2.5

Data From Oscillogram No. 6.

t sec.	i	R ex.	R total	E
.05	.439	15.5	75.3	212.8
.1	.690	"	"	"
.2	1.067	"	"	"
.3	1.403	"	"	"
.4	1.758	"	"	"
.5	2.070	"	"	"
.6	2.323	"	"	"
.7	2.490	"	"	"
.8	2.615	"	"	"
.9	2.720	"	"	"
1.0	2.785	"	"	"
1.1	2.800	"	"	"
1.2	2.808	"	"	"
1.3	2.825	"	"	"
1.4	2.825	"	"	"
1.5	2.825	"	"	"

Calculated Data, Oscillogram No. 6.

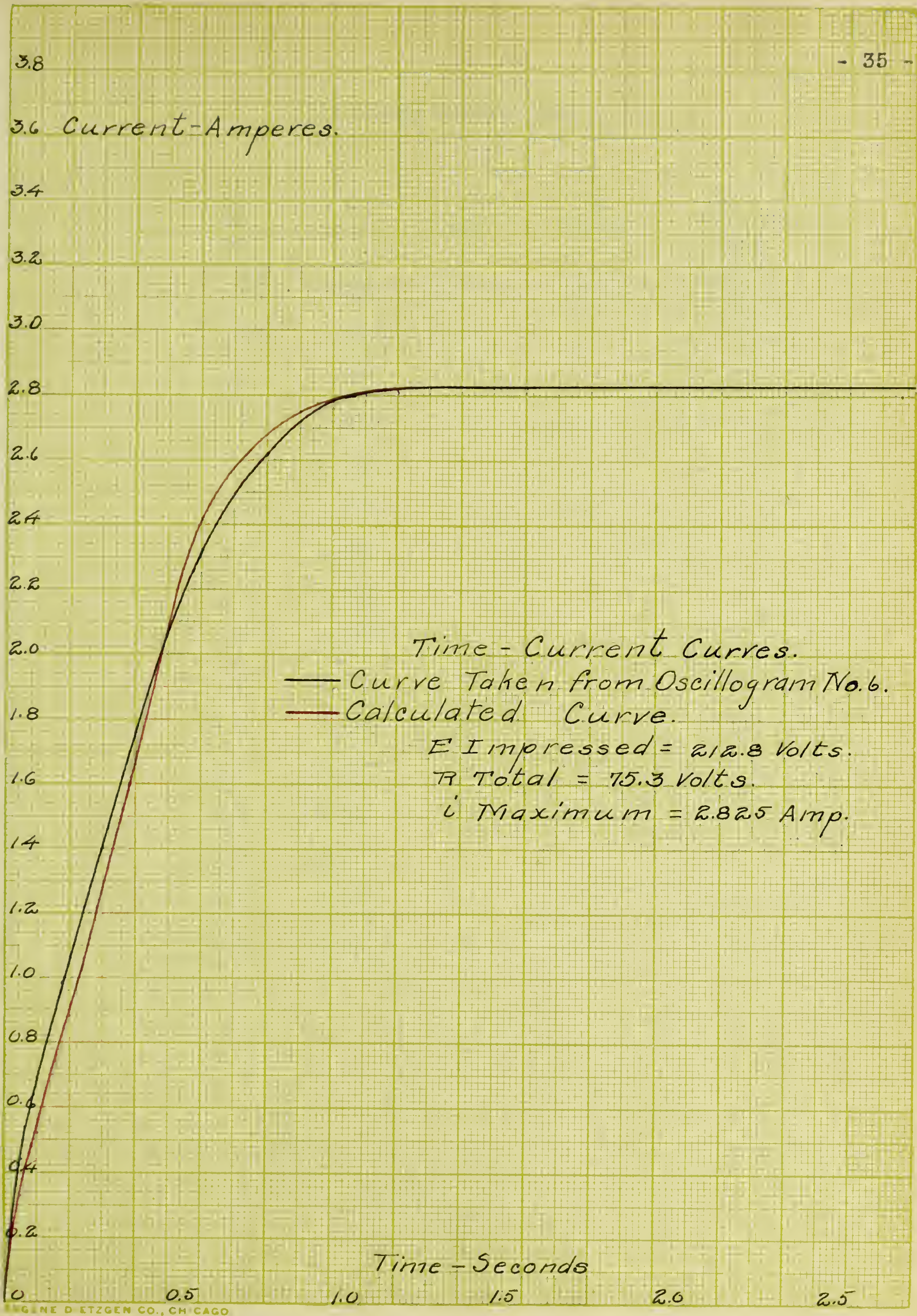
E = 212.8 volts R = 75.3 ohms N = 6000 turns

Assume $\triangle t$ = .05 seconds

$$\text{Then, } E - i\,R = \frac{N}{.5 \times 10^7} \triangle\varphi = 212.8 - 75.3 = 75.3\,(2.813 - i)$$

$$\triangle\varphi = \frac{75.3 \times .5 \times 10^7}{6000}\,(2.813 - i) = .0628 \times 10^6\,(2.813 - i)$$

t	$\triangle\varphi$	$\Sigma\triangle\varphi$	i	(2.813 - i)	$\triangle\varphi$	$\Sigma\triangle\varphi$	i
.05	.1768	.1768	.37	2.443	.1535	.1535	.33
.10	.1535	.307	.53	2.283	.1434	.2969	.52
.15	.1434	.440	.69	2.123	.1334	.4303	.68
.20	.1334	.564	.86	1.953	.1228	.5531	.84
.25	.1228	.676	1.03	1.783	.1120	.6651	1.01
.30	.1120	.777	1.22	1.593	.1001	.7652	1.19
.35	.1001	.865	1.44	1.373	.0862	.8514	1.39
.40	.0862	.938	1.72	1.083	.0687	.9201	1.66
.45	.0687	.989	1.97	.843	.0530	.9731	1.88
.50	.0530	1.026	2.18	.633	.0398	1.0129	2.09
.55	.0398	1.053	2.36	.453	.0285	1.0414	2.27
.60	.0285	1.069	2.49	.323	.0203	1.0617	2.42
.65	.0203	1.082	2.57	.243	.0153	1.0770	2.52
.70	.0153	1.092	2.64	.173	.0109	1.0880	2.60
.75	.0109	1.099	2.71	.103	.0065	1.095	2.67
.80	.0065	1.102	2.73	.083	.0052	1.1002	2.72
.85	.0052	1.105	2.76	.053	.0033	1.1035	2.74
.90	.0033	1.107	2.78	.033	.0021	1.1056	2.75
.95	.0021	1.108	2.79	.023	.0014	1.1070	2.78


Current-Amperes.
3.8
3.6
3.4
3.2
3.0
2.8
2.6
2.4
2.2
2.0
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0
0.5
1.0
1.5
2.0
2.5
Time - Seconds
Time - Current Curves.
Curve Taken from Oscillogram No. 6.
Calculated Curve.
E Impressed = 212.8 Volts.
R Total = 75.3 Volts.
i Maximum = 2.825 Amp.

Data From Oscillogram No. 7.

t sec.	i	R ex.	R total	E imp.
.05	.410	33.72	93.52	216
.1	.640	"	"	"
.2	.995	"	"	"
.3	1.276	"	"	"
.4	1.549	"	"	"
.5	1.778	"	"	"
.6	1.945	"	"	"
.7	2.070	"	"	"
.8	2.153	"	"	"
.9	2.220	"	"	"
1.0	2.260	"	"	"
1.1	2.300	"	"	"
1.2	2.31	"	"	"
1.3	2.31	"	"	"
1.4	2.31	"	"	"
1.5	2.31	"	"	"

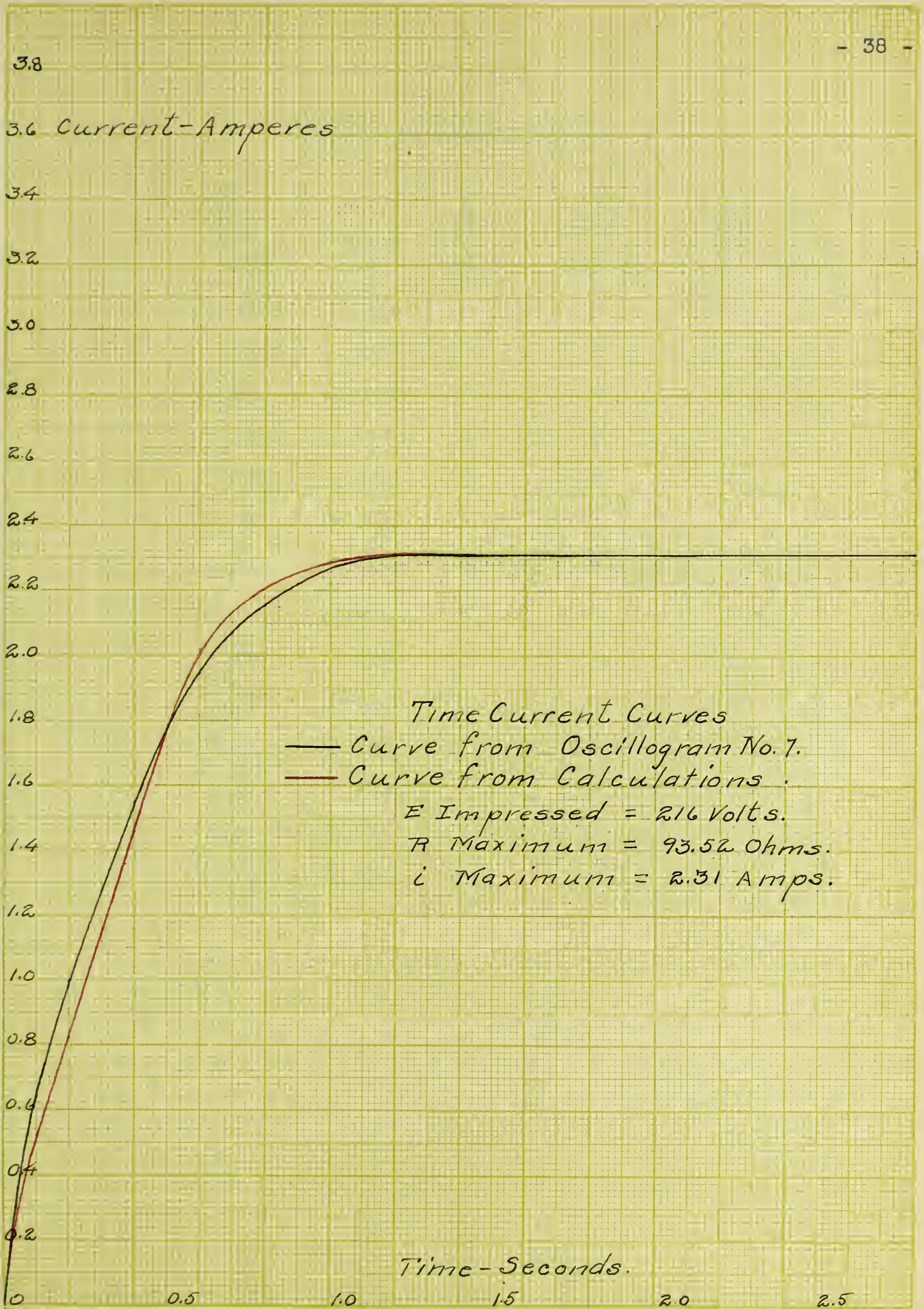
Calculated Data, Oscillogram No. 7.

E = 216 volts R = 93.52 ohms N = 6000 turns

$$E - i R = \frac{N}{.5 \times 10^7} \cdot \triangle\varphi = 216 - 93.52\, i = 93.52\,(2.31 - i)$$

$$\triangle\varphi = \frac{.5 \times 10^7 \times 93.52}{6000}\,(2.31 - i) = .0779 \times 10^6\,(2.31 - i)$$

t	$\triangle\varphi$	$\Sigma\triangle\varphi$	i	(2.31 - i)	$\triangle\varphi$	$\Sigma\triangle\varphi$	i
.05	.180	.180	.37	1.94	.1513	.1513	.33
.10	.1513	.303	.53	1.78	.1389	.2902	.52
.15	.1389	.429	.68	1.63	.1272	.4174	.67
.20	.1272	.545	.83	1.48	.1155	.5329	.82
.25	.1155	.648	.98	1.33	.1038	.6367	.96
.30	.1038	.740	1.14	1.17	.0912	.7279	1.12
.35	.0912	.819	1.31	1.00	.0779	.8058	1.28
.40	.0779	.884	1.50	.81	.0632	.8690	1.46
.45	.0632	.932	1.69	.62	.0483	.9173	1.63
.50	.0483	.966	1.85	.46	.0359	.9532	1.79
.55	.0359	.989	1.97	.34	.0265	.9797	1.92
.60	.0265	1.006	2.06	.25	.0195	.9992	2.02
.65	.0195	1.019	2.14	.17	.0133	1.0125	2.10
.70	.0133	1.026	2.18	.13	.0101	1.0226	2.16
.75	.0101	1.033	2.22	.08	.0062	1.0288	2.20
.80	.0062	1.035	2.25	.06	.0047	1.0335	2.23
.85	.0047	1.038	2.26	.05	.0039	1.0374	2.25
.90	.0039	1.041	2.28	.03	.0023	1.0397	2.27
.95	.0023	1.042	2.29	.02	.0016	1.0413	2.28
1.00	.0016	1.043	2.30	.01	.0008	1.0421	2.29


Current-Amperes
3.8
3.6
3.4
3.2
3.0
2.8
2.6
2.4
2.2
2.0
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
Time Current Curves
Curve from Oscillogram No. 7.
Curve from Calculations.
E Impressed = 216 Volts.
R Maximum = 93.52 Ohms.
i Maximum = 2.31 Amps.
Time - Seconds.
0
0.5
1.0
1.5
2.0
2.5

Data From Oscillogram No. 8.

t sec.	i	R ex.	R total	E imp.
.05	.397	65.2	126.0	217
.10	.585	"	"	"
.20	.877	"	"	"
.30	1.090	"	"	"
.40	1.255	"	"	"
.50	1.402	"	"	"
.60	1.507	"	"	"
.70	1.590	"	"	"
.80	1.640	"	"	"
1.00	1.695	"	"	"
1.20	1.715	"	"	"
1.40	1.730	"	"	"
1.60	1.735	"	"	"
1.80	1.735	"	"	"
2.00	1.735	"	"	"

Calculated Data, Oscillogram No. 8.

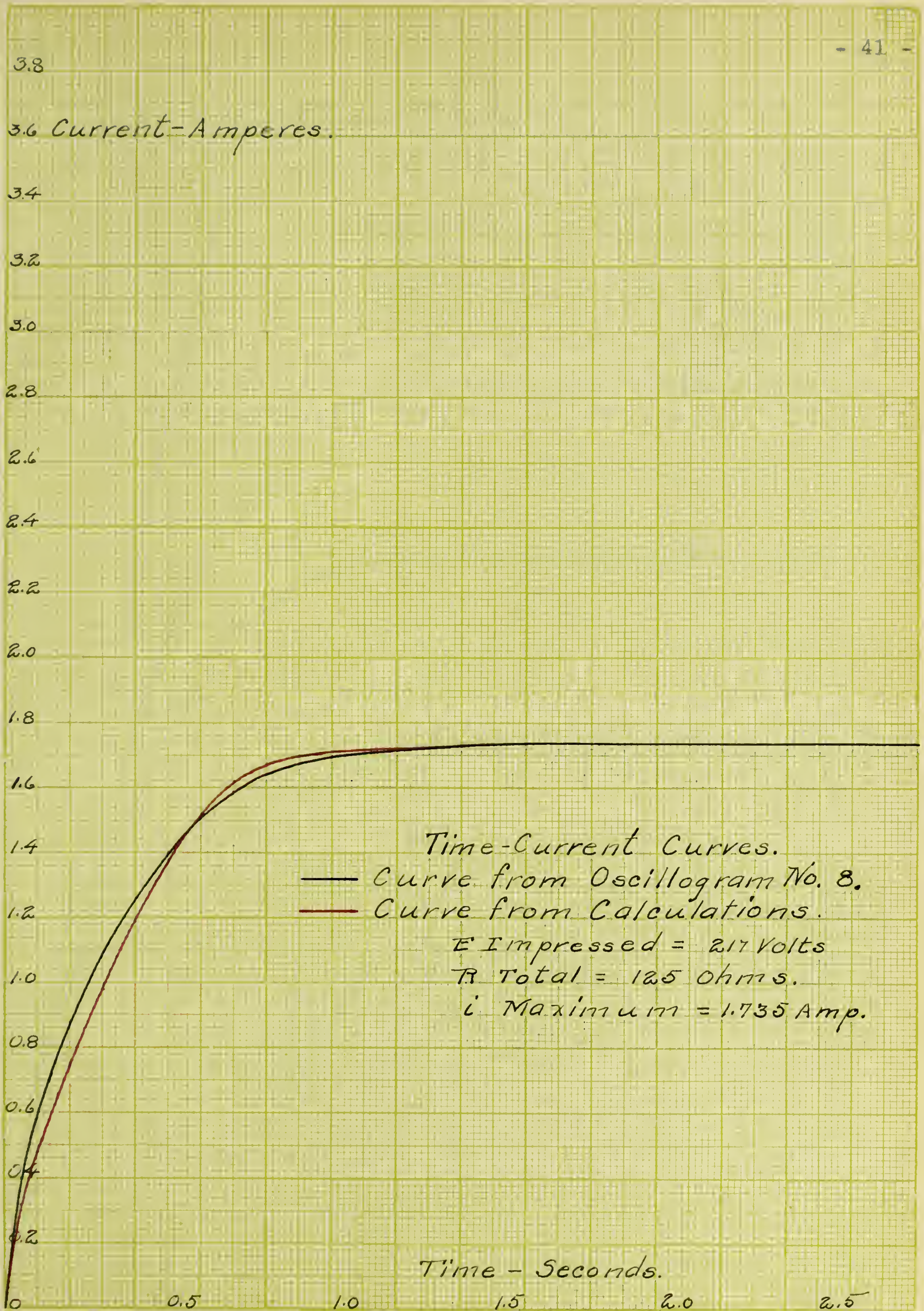
E = 217 volts R = 125 ohms N = 6000 turns

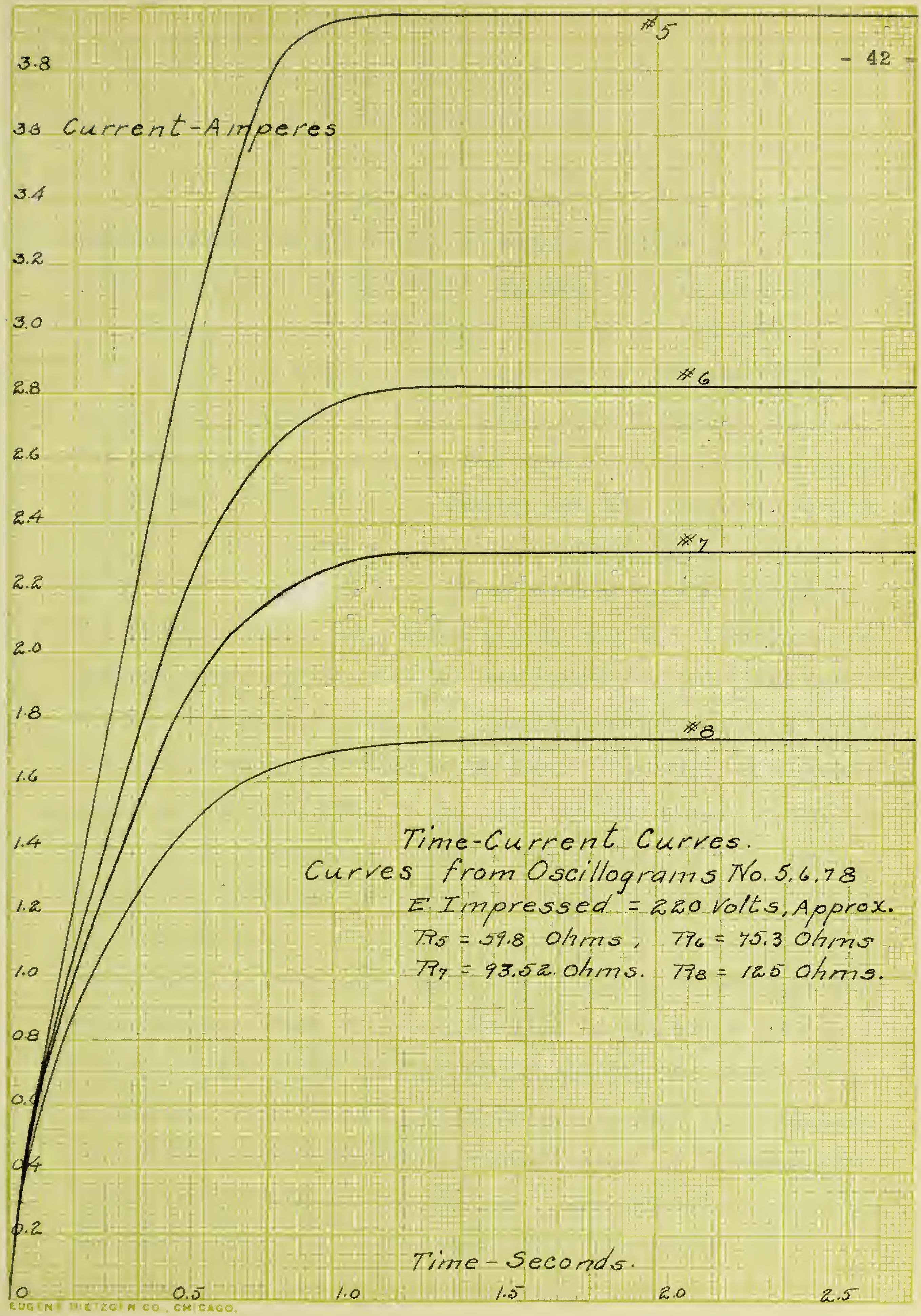
Assume $\triangle t$ = .05 seconds

Then, $E - i R = \frac{N}{.5 \times 10^7} \cdot \triangle \varphi = 217 - 125 i = 125 (1.735 - i)$

$$\triangle \varphi = \frac{125 \times .5 \times 10^7}{6000} = (1.735 - i) = .1042 \times 10^6 (1.735 - i)$$

t	$\triangle \varphi$	$\Sigma \triangle \varphi$	i	(1.735 - i)	$\triangle \varphi$	$\Sigma \triangle \varphi$	i
.05	.181	.181	.38	1.355	.1413	.1413	.32
.10	.1413	.283	.51	1.225	.1276	.2689	.48
.15	.1276	.396	.64	1.095	.1141	.3830	.62
.20	.1141	.497	.77	.965	.1006	.4836	.75
.25	.1006	.584	.89	.845	.0880	.5716	.87
.30	.0880	.660	1.00	.735	.0766	.6482	.98
.35	.0766	.725	1.12	.615	.0642	.7124	1.09
.40	.0642	.777	1.21	.521	.0542	.7666	1.19
.45	.0542	.821	1.31	.425	.0443	.8109	1.29
.50	.0443	.855	1.41	.325	.0339	.8448	1.38
.55	.0339	.879	1.48	.255	.0266	.8714	1.46
.60	.0266	.898	1.55	.185	.0193	.8907	1.52
.65	.0193	.910	1.60	.135	.0141	.9048	1.56
.70	.0141	.919	1.63	.105	.0109	.9157	1.62
.75	.0109	.927	1.67	.065	.0068	.9225	1.65
.80	.0068	.929	1.68	.055	.0057	.9282	1.67
.85	.0057	.934	1.70	.035	.0036	.9318	1.68
.90	.0036	.935	1.71	.025	.0026	.9344	1.70
.95	.0026	.937	1.72	.015	.0016	.9360	1.71


Current-Amperes.
3.8
3.6
3.4
3.2
3.0
2.8
2.6
2.4
2.2
2.0
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0
Time-Current Curves.
Curve from Oscillogram No. 8.
Curve from Calculations.
E Impressed = 217 Volts
R Total = 125 Ohms.
i Maximum = 1.735 Amp.
Time - Seconds.
0.5
1.0
1.5
2.0
2.5


Current-Amperes
3.8
3.6
3.4
3.2
3.0
2.8
2.6
2.4
2.2
2.0
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0
#5
#6
#7
#8
Time-Current Curves.
Curves from Oscillograms No. 5.6.7.8
E Impressed = 220 Volts, Approx.
R5 = 59.8 Ohms, R6 = 75.3 Ohms
R7 = 93.52 Ohms. R8 = 125 Ohms.
Time-Seconds.
0.5
1.0
1.5
2.0
2.5
EUGENE DIETZGEN CO., CHICAGO.

--- VI DATA AND CALCULATIONS ---

It will be noticed that the theoretical and experimental curves for 110 volts impressed correspond very closely while the curves for 220 volts impressed diverge considerably. It will also be noticed, however, for the 110 volt curves, that as the resistance becomes less making the inductance figure more strongly that the curves deviate farther apart. The same thing is seen to be true even in a more marked degree in the case of the 220 volt curves. As the resistance becomes less the curves deviate farther and farther apart. This would seem to indicate that the error lies in the inductance. Furthermore, as has been shown before, the line resistance is inappreciable and the rheostat resistance, in each case and the field resistance were all measured carefully and averaged and, furthermore, the same error appears to creep into every case, all of which goes to show that the resistance is not in error to any appreciable degree. Again, the greatest deviation comes on the lower part of the curves where the resistance is less appreciable and where the inductance figures in a greater degree. At this point the theoretical curves fall below the experimental. That is, over this part of the theoretical curves, the successive values of the current are too low, which seems to indicate that the saturation curve, as actually determined, is too high; due probably to a small amount of residual magnetism in the field of the machine. The attempt to discharge the field before taking the saturation curve data was, in all

probability, not entirely fruitful of results.